



# An improved harmony search algorithm for power economic load dispatch

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## ABSTRACT

A meta-heuristic algorithm called harmony search (HS), mimicking the improvisation process of music players, has been recently developed. The HS algorithm has been successful in several optimization problems. The HS algorithm does not require derivative information and uses stochastic random search instead of a gradient search. In addition, the HS algorithm is simple in concept, few in parameters, and easy in implementation. This paper presents an improved harmony search (IHS) algorithm based on exponential distribution for solving economic dispatch problems. A 13-unit test system with incremental fuel cost function taking into account the valve-point loading effects is used to illustrate the effectiveness of the proposed IHS method. Numerical results show that the IHS method has good convergence property. Furthermore, the generation costs of the IHS method are lower than those of the classical HS and other optimization algorithms reported in recent literature.

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## 1. Introduction

The economic dispatch problem (EDP) is related to the optimum generation scheduling of available generators in a power system to minimize the total fuel cost while satisfying the load demand and operational constraints. EDP plays an important role in operation planning and control of modern power systems [1].

Over the past few years, a number of approaches have been developed for solving the EDP using classical mathematical programming methods [2–8]. Meanwhile, classical optimization methods are highly sensitive to starting points and frequently converge to local optimum solution or diverge altogether. Linear programming methods are fast and reliable but the main disadvantage associated with the piecewise linear cost approximation. Nonlinear programming methods have a problem of convergence and algorithmic complexity. Newton based algorithm have a problem in handling large number of inequality constraints [9].

Recently, in order to make numerical methods more convenient for solving the EDPs, modern optimization techniques [10–15] have been successfully employed to solve the EDP as a non-smooth optimization problem. A global optimization technique known as the harmony search (HS) is one of these modern techniques [16]. HS algorithm proposed in [17] has been recently developed in an analogy with music improvisation process where musicians in an ensemble continue to polish their pitches in order to obtain better harmony. Jazz improvisation seeks to find musically pleasing har-

mony similar to the optimum design process which seeks to find optimum solution. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is determined by the set of values assigned to each decision variable [18]. In addition, HS uses a stochastic random search instead of a gradient search so that derivative information is unnecessary. However, recent studies [19–21] have identified some deficiencies related to the premature convergence in the performance of classical HS.

In this paper, we propose a novel approach for solving the EDP using an improved harmony search (IHS) algorithm. An EDP based on a 13-unit test system [22] with incremental fuel cost function taking into account the valve-point loading effects is employed to demonstrate the performance of the IHS. The valve-point loading effects introduce multiple minima in the solution space. Numerical results obtained with the proposed IHS approach were compared with classical HS method and other optimization results reported in literature.

The remainder of this paper is organized as follows. Section 2 explains the formulation of the EDP. In Sections 3 and 4, the classical HS and the proposed IHS are described. Simulation results of HS and IHS are presented and compared with those of other algorithms in Section 5. Lastly, Section 6 outlines our conclusions.

## 2. Economic dispatch

The primary concern of an EDP is to minimize the total fuel cost at thermal power plants subjected to the operating constraints of a power system. Therefore, it can be formulated mathematically

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with an objective function and two constraints. The equality and inequality constraints are represented by [23]:

$$\sum_{i=1}^n P_i - P_L - P_D = 0 \quad (1)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (2)$$

In the power balance criterion, an equality constraint must be satisfied, as shown in Eq. (1). The generated power should be the same as the total load demand plus total line losses. The generating power of each generator should lie between maximum and minimum limits represented by Eq. (2), where  $P_i$  is the power of generator  $i$  (in MW);  $n$  is the number of generators in the system;  $P_D$  is the system's total demand (in MW);  $P_L$  represents the total line losses (in MW) and  $P_i^{\min}$  and  $P_i^{\max}$  are, respectively, the output of the minimum and maximum operation of the generating unit  $i$  (in MW). The total fuel cost function is formulated as follows [23]:

$$\min f = \sum_{i=1}^n F_i(P_i) \quad (3)$$

where  $F_i$  is the total fuel cost for the generator unity  $i$  (in \$/h), which is defined by equation:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (4)$$

where  $a_i$ ,  $b_i$  and  $c_i$  are cost coefficients of generator  $i$ .

The sequential valve-opening process for multi-valve steam turbines produces ripple like effect in the heat rate curve of the generator. This effect is included in EDP by superimposing the basic quadratic fuel cost characteristics with a rectified sinusoidal component. In this context, Eq. (4) can be modified as:

$$\tilde{F}_i(P_i) = F_i(P_i) + |e_i \sin(f_i(P_i^{\min} - P_i))| \quad \text{or} \quad (5)$$

$$\tilde{F}_i(P_i) = a_i P_i^2 + b_i P_i + c_i + |e_i \sin(f_i(P_i^{\min} - P_i))| \quad (6)$$

where  $e_i$  and  $f_i$  are valve-point loading coefficients of generator  $i$ . Hence, the total fuel cost that must be minimized, according to Eq. (3), is modified to:

$$\min f = \sum_{i=1}^n \tilde{F}_i(P_i) \quad (7)$$

where  $\tilde{F}_i$  is the cost function of generator  $i$  (in \$/h) defined by Eq. (6). In the case study presented here, we disregarded the transmission losses,  $P_L$  (mentioned in Eq. (1)), i.e., in this work  $P_L = 0$ .

### 3. Harmony search to solve the economic dispatch problem

This section presents a brief overview of the HS. After, in Section 4, the modification procedure of the proposed IHS algorithm is detailed.

#### 3.1. Classical harmony search algorithm

Recently, Geem et al. [17] proposed a new HS meta-heuristic algorithm that was inspired by musical process of searching for a perfect state of harmony. The harmony in music is analogous to the optimization solution vector, and the musician's improvisations are analogous to local and global search schemes in optimization techniques [24]. The HS algorithm does not require initial values for the decision variables. Furthermore, instead of a gradient search, the HS algorithm uses a stochastic random search that is based on the harmony memory considering rate and the pitch adjusting rate so that derivative information is unnecessary.

In the HS algorithm, musical performances seek a perfect state of harmony determined by aesthetic estimation, as the optimiza-

tion algorithms seek a best state (i.e. global optimum) determined by objective function value. It has been successfully applied to various optimization problems in computation and engineering fields [17–21,23–28].

The optimization procedure of the HS algorithm consists of steps 1–5, as follows:

Step 1: Initialize the optimization problem and algorithm parameters.

Step 2: Initialize the harmony memory (HM).

Step 3: Improvise a new harmony from the HM.

Step 4: Update the HM.

Step 5: Repeat Steps 3 and 4 until the termination criterion has been satisfied.

The detailed explanation of these steps can be found in [17,18,24] which are summarized in the following:

*Step 1. Initialize the optimization problem and HS algorithm parameters.* First, the optimization problem is specified as follows:

$$\text{Minimize } f(x) \text{ subject to } x_i \in X_i, \quad i = 1, \dots, N$$

where  $f(x)$  is the objective function,  $x$  is the set of each decision variable ( $x_i$ );  $X_i$  is the set of the possible range of values for each design variable (continuous design variables), that is,  $x_{i,lower} \leq x_i \leq x_{i,upper}$ , where  $x_{i,lower}$  and  $x_{i,upper}$  are the lower and upper bounds for each decision variable; and  $N$  is the number of design variables. In this context, the HS algorithm parameters that are required to solve the optimization problem are also specified in this step. The number of solution vectors in harmony memory (HMS) that is the size of the harmony memory matrix, harmony memory considering rate (HMCR), pitch adjusting rate (PAR), and the maximum number of searches (stopping criterion) are selected in this step. Here, HMCR and PAR are parameters that are used to improve the solution vector. In this context, both are defined in Step 3.

*Step 2. Initialize the harmony memory (HM).* The harmony memory (HM) is a memory location where all the solution vectors (sets of decision variables) are stored. In Step 2, the HM matrix, shown in Eq. (8), is filled with randomly generated solution vectors using a uniform distribution, where

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_{N-1}^1 & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_{N-1}^2 & x_N^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \dots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \dots & x_{N-1}^{HMS} & x_N^{HMS} \end{bmatrix} \quad (8)$$

*Step 3. Improvise a new harmony from the HM.* A new harmony vector,  $x' = (x'_1, x'_2, \dots, x'_N)$ , is generated based on three rules: (i) memory consideration, (ii) pitch adjustment, and (iii) random selection. Generating a new harmony is called 'improvisation'.

In the memory consideration, the value of the first decision variable ( $x'_1$ ) for the new vector is chosen from any value in the specified HM range ( $x_1 - x_1^{HMS}$ ). Values of the other decision variables ( $x'_2, \dots, x'_N$ ) are chosen in the same manner. The HMCR, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in the HM, while  $(1 - HMCR)$  is the rate of randomly selecting one value from the possible range of values.

$$x'_i \leftarrow \begin{cases} x'_i \in \{x_1^1, x_2^1, \dots, x_1^{HMS}\} & \text{with probability HMCR} \\ x'_i \in X_i & \text{with probability}(1 - HMCR) \end{cases} \quad (9)$$

After, every component obtained by the memory consideration is examined to determine whether it should be pitch-adjusted. This operation uses the PAR parameter, which is the rate of pitch adjustment as follows:

$$\text{Pitch adjusting decision for } x'_i \begin{cases} \text{Yes with probability PAR} \\ \text{No with probability}(1-\text{PAR}). \end{cases} \quad (10)$$

The value of  $(1 - \text{PAR})$  sets the rate of doing nothing. If the pitch adjustment decision for  $x'_i$  is Yes, then  $x'_i$  is replaced as follows:

$$x'_i \leftarrow x'_i \pm r \cdot bw, \quad (11)$$

where  $bw$  is an arbitrary distance bandwidth,  $r$  is a random number generated using uniform distribution between 0 and 1. In Step 3, HM consideration, pitch adjustment or random selection is applied to each variable of the new harmony vector in turn.

**Step 4. Update the HM.** If the new harmony vector,  $x' = (x'_1, x'_2, \dots, x'_N)$  is better than the worst harmony in the HM, judged in terms of the objective function value, the new harmony is included in the HM and the existing worst harmony is excluded from the HM.

**Step 5.** Repeat Steps 3 and 4 until the termination criterion has been satisfied.

#### 4. Improved HS algorithm

HS is good at identifying the high performance regions of the solution space at a reasonable time, but gets into trouble in performing local search for numerical applications. In order to improve the performance of the HS algorithm and eliminate the drawbacks lie with fixed values of HMCR and PAR, Mahdavi et al. [19] proposed an improved harmony search algorithm that uses variable PAR and  $bw$  in improvisation step. Also, Omran and Mahdavi [20] proposed a new variant of harmony search, called the global best harmony search, in which concepts from swarm intelligence are borrowed to enhance the performance of HS such that the new harmony can mimic the best harmony in the HM.

The IHS proposed in this work has exactly the same steps of classical HS with exception that Step 3, where the IHS dynamically updates PAR. In this case, PAR is update as follows:

$$\text{PAR}(t) = \frac{1}{\text{HMS} \cdot N}, \quad (12)$$

where  $\text{PAR}(t)$  is the pitch adjusting rate for generation  $t$ . In addition, the Eq. (11) is changed by generation of  $x'_i$  using an exponential probability distribution with density function given by [29]:

$$f(y) = \frac{1}{2b} \exp\left(\frac{-|y-a|}{b}\right), \quad -\infty \leq y \leq \infty \quad \text{with } a, b > 0 \quad (13)$$

It is evident that one can control the variance by changing the parameters  $a$  and  $b$ . Generating random numbers using exponential distribution sequences may provide a good compromise between the probability of having a large number of small amplitudes around the current points (fine tuning) and a small probability of having higher amplitudes, which may allow particles to move away from the current point and escape from local minima [29].

In the HIS, the  $x'_i$  is given by:

$$x'_i \leftarrow x'_i + e \cdot bw, \quad (14)$$

where  $e$  is a random number generated with truncated exponential distribution in range  $[-1, 1]$  with  $a = 0.30$  and  $b = 1$ . Eq. (14) is in-

spired in works about the tuning of control parameters in evolutionary algorithms based on probability distributions (see details in [29–32]).

#### 5. Case study of 13 thermal units and analysis of optimization results

This case study consisted of 13 thermal units of generation with the valve-point effects, as given in Table 1. The system data shown in Table 1 is also available in [22,33]. In this case, the load demand expected to be determined was  $P_D = 1800$  MW.

Each optimization method was implemented in Matlab (Math-Works). All the programs were run on a 3.2 GHz Pentium IV processor with 2 GB of random access memory. In each case study, 50 independent runs were made for each of the optimization methods involving 50 different initial trial solutions for each optimization method.

The total number of solution vectors in classical HS, i.e., the HMS, was 15, and the HMCR and the PAR were 0.85 and 0.45, respectively. In IHS, the setup were HMS, was 15 and the HMCR was 0.85. In this paper, the optimization approaches are adopted using 22,500 cost function evaluations in each run.

A key factor in the application of optimization methods is how the algorithm handles the constraints relating to the problem. In this work, a penalty-based method inspired in [34] was used. In this context, to avoid the violation of equality constraint given by Eq. (1) of the power balance criterion, a repair process is applied to each solution in order to guarantee that a generated solution by HS or IHS is feasible. The adopted procedure here is presented in Fig. 1. After the application of the repair procedure the solution given by HS or IHS approaches can be evaluated by Eq. (7).

Numerical results obtained for this case study are given in Table 2, which shows that the IHS has both a better economic cost and lower mean cost than the classical HS. The best results obtained for solution vector  $P_i$ ,  $i = 1, \dots, 13$  with IHS with minimum cost of 17960.3661 \$/h is given in Table 3. Table 4 compares the results obtained in this paper with those of other studies reported in the literature. Note that in studied case, the best result reported here using IHS is comparatively lower than recent studies presented in literature.

#### 6. Conclusions and future research

Economic dispatch is an important function in the power system operation. Different techniques have been reported in the literature pertaining to EDP [1–15]. In this study, application of HS and IHS algorithms to solve EDPs has been investigated.

**Table 1**  
Data for the 13 thermal units.

Thermal unit	$P_i^{\min}$	$P_i^{\max}$	A	b	c	E	f
1	0	680	0.00028	8.10	550	300	0.035
2	0	360	0.00056	8.10	309	200	0.042
3	0	360	0.00056	8.10	307	150	0.042
4	60	180	0.00324	7.74	240	150	0.063
5	60	180	0.00324	7.74	240	150	0.063
6	60	180	0.00324	7.74	240	150	0.063
7	60	180	0.00324	7.74	240	150	0.063
8	60	180	0.00324	7.74	240	150	0.063
9	60	180	0.00324	7.74	240	150	0.063
10	40	120	0.00284	8.60	126	100	0.084
11	40	120	0.00284	8.60	126	100	0.084
12	55	120	0.00284	8.60	126	100	0.084
13	55	120	0.00284	8.60	126	100	0.084

```

Randomly select a component  $i$  of  $j$ -th solution of HM
If  $\sum_{i=1}^n P_i < P_D$ 
    Add an amount  $w \leftarrow \left| \sum_{i=1}^n P_i - P_D \right|$  to  $x_i^j$  that doesn't violate  $P_i^{\max}$ , such as  $x_i^j \leftarrow \min(\text{minimum}(x_i^j + w, P_i^{\max}))$ 
Else If  $\sum_{i=1}^n P_i > P_D$ 
    Subtract an amount  $w \leftarrow \left| \sum_{i=1}^n P_i - P_D \right|$  to  $x_i^j$  that doesn't violate  $P_i^{\min}$ , such as  $x_i^j \leftarrow \max(\text{maximum}(x_i^j - w, P_i^{\min}))$ 
End if
End while
    
```

Fig. 1. Constraints handling used in HS and IHS approaches.

Table 2

Convergence results (50 runs) of a case study of 13 thermal units with valve-point and  $P_D = 1800$  MW.

Optimization method	Maximum cost (\$/h)	Minimum cost (\$/h)	Mean cost (\$/h)	Standard deviation (\$/h)
HS	18070.1762	17965.6204	17986.5626	26.3702
IHS	17971.6512	17960.3661	17965.4152	16.9531

Table 3

Best result (50 runs) obtained for the case study using IHS.

Power	Generation (MW)
$P_1$	628.3185
$P_2$	149.5994
$P_3$	222.7491
$P_4$	109.8666
$P_5$	60.0000
$P_6$	109.8666
$P_7$	109.8666
$P_8$	109.8666
$P_9$	109.8666
$P_{10}$	40.0000
$P_{11}$	40.0000
$P_{12}$	55.0000
$P_{13}$	55.0000
$\sum_{i=1}^{13} P_i$	1800.0000

Table 4

Comparison of results for EDP with 13 thermal units.

Optimization technique	Case study with 13 thermal units
Cultural differential evolution [35]	17963.94
Chaotic differential evolution with sequential quadratic programming [36,37]	17963.94
Chaotic particle swarm optimization [38]	17963.96
Differential evolution [34]	17963.83
Genetic algorithm based on differential evolution [39]	17963.83
Hybrid differential evolution [40]	17975.73
Hybrid evolutionary programming with sequential quadratic programming [41]	17991.03
Hybrid genetic algorithm [42]	17992.92
Improved evolutionary programming [22]	17994.07
Improved genetic algorithm [43]	17963.98
Particle swarm optimization [41]	18030.72
Particle swarm optimization with sequential quadratic programming [41]	17969.93
Pattern search method [44]	17969.17
Quantum particle swarm optimization [23]	17963.95
Self-tuning hybrid differential evolution [40]	17963.79
Best result of this paper using IHS	17960.3661

The IHS algorithms ability has been demonstrated using an illustrative example consisting of 13 thermal units whose incremental fuel cost function takes into account the valve-point loading effects. Moreover, in order to handle constraints effectively, a constraint treatment mechanism inspired in [34] is devised in calculus of cost function used in HS and IHS approaches. Numerical results reveal that the IHS algorithm converged to good solutions in comparison with results using HS and results of recent literature.

In future work, we plan to study the HS algorithms in multiobjective EDPs with units having prohibited zones and valve-point loading effects.

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